Telkom University

Diploma of Telecommunication Engineering

DTH1B3 - MATEMATIKA TELEKOMUNIKASI I

Vektor

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Capaian Pembelajaran

- [C4, A2] Mampu memahami vektor-vektor pada ruang dua dan ruang tiga serta operasi-operasi yang berlaku.

Materi Pembelajaran

1. Scalar dan Vektor
2. Operasi vektor
3. Sistem Koordinat 3D
4. Operasi Vektor dalam Sistem Koordinat

Pengertian : Skalar & Vektor

Skalar : Besaran yang hanya memiliki nilai

Contoh : Temperatur, jarak, Berat

Vektor : Besaran yang memiliki nilai dan arah

Contoh : Medan listrik, medan magnet, Gaya, kecepatan, percepatan, dll

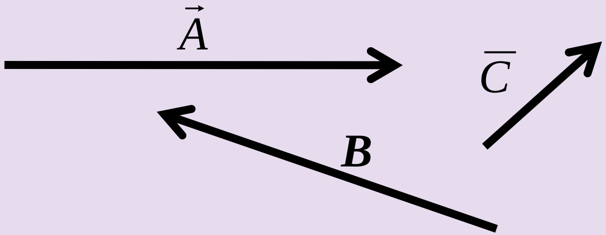
Pengertian : Skalar & Vektor

Manakah dari besaran berikut yang skalar dan manakah yang vektor

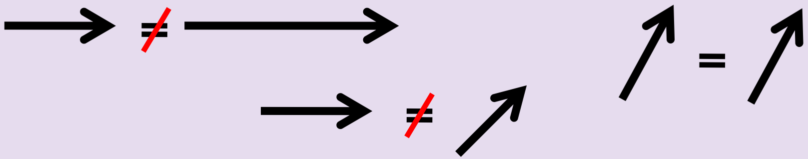
a) percepatan pesawat saat tinggal landas
b) Jumlah penumpang dalam pesawat
c) Lama penerbangan
d) Tujuan penerbangan
e) Jumlah bahan bakar yang dibutuhkan pesawat

Representasi Vektor

Secara simbolik kita dapat merepresentasikan sebuah vector quantity sebagai sebuah **panah** :



Dua buah vektor dikatakan sama jika kedua vektor tersebut memiliki magnitudo dan arah yang identik :



a. **Panjang** dari sebuah panah proposional dengan **magnitudo** vector quantity.
b. **Orientasi dari panah** mengindikasikan **arah** dari vector quantity.
c. Penamaan variabel pada vector quantity akan selalu **dicetak tebal** dan disertai **overbar**.

Representasi Vektor

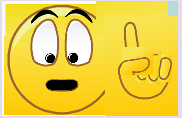
$$\vec{A} = \left|\vec{A}\right| \hat{a}_A$$

Dengan :

$\left|\vec{A}\right|$ adalah besar vektor A atau panjang vektor A (**magnitude** vektor A)

$\hat{a}_A$ adalah unit vektor A atau **vektor satuan** searah A

Vektor satuan atau unit vektor menyatakan **arah vektor**, besarnya **satu.**



$$\hat{a}_A = \frac{\vec{A}}{\left|\vec{A}\right|}$$

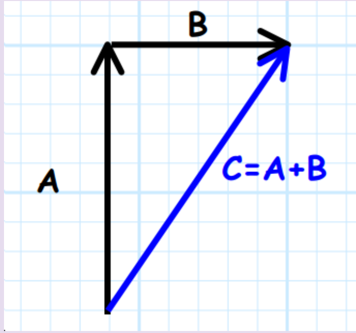
$$\left|\hat{a}_A\right| = 1$$

Operasi Vektor

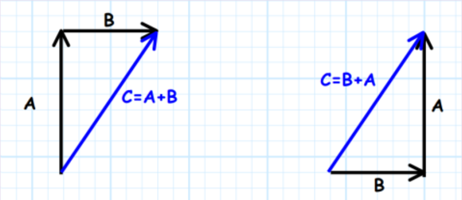
Jika kita mengetahui aturan dari operasi vektor, kita dapat menganalisa, memanipulasi, dan membuat operasi vektor menjadi lebih sederhana.

Penjumlahan Vektor

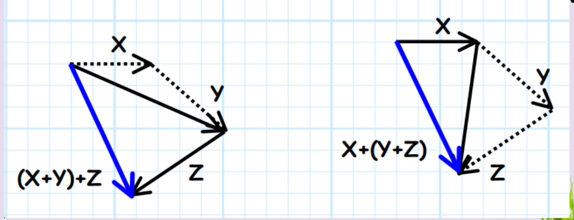
$$\vec{A} + \vec{B} = \vec{C}$$



1. Vector addition is **commutative-> A+B = B+A**

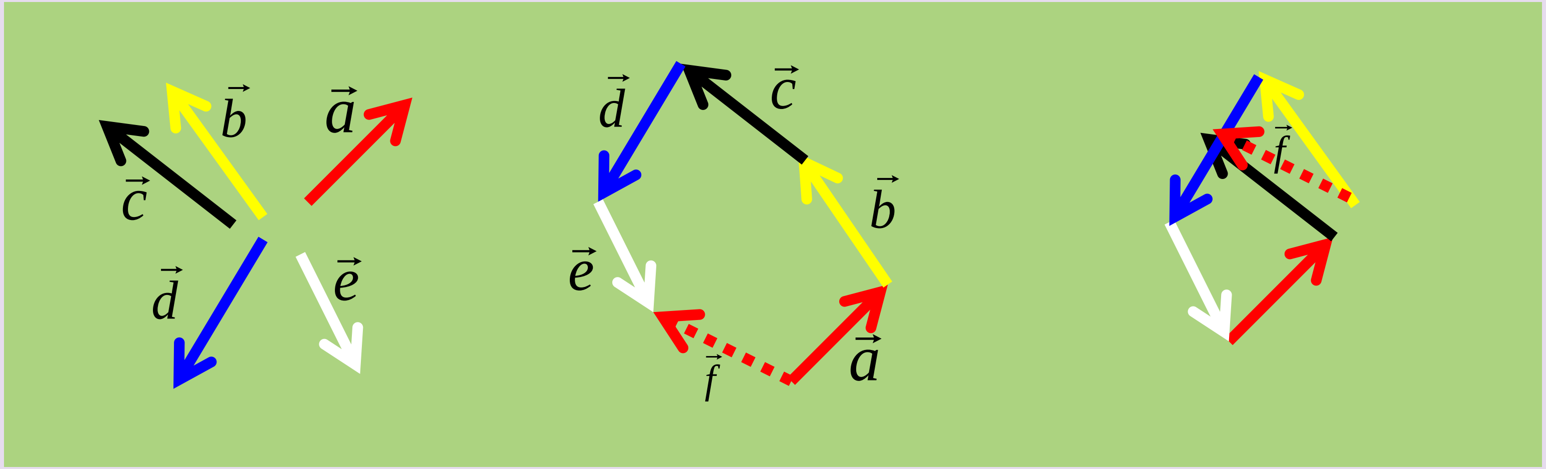


2. Vector addition is **associative-> (X+Y) + Z = X + (Y+Z)**



Operasi Vektor

Penjumlahan Vektor



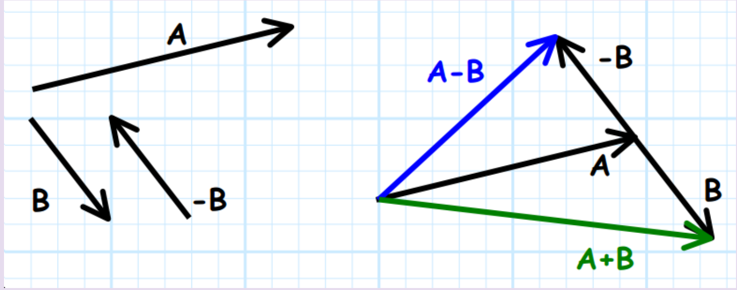
$$\vec{f} = \vec{a} + \vec{b} + \vec{c} + \vec{d} + \vec{e} \qquad \vec{f} = \vec{b} + \vec{d} + \vec{e} + \vec{a} + \vec{c}$$

Operasi Vektor

Pengurangan Vektor

the negative of a vector is a vector with **equal magnitude** but **opposite direction.**

$$A + (-B) = A - B$$

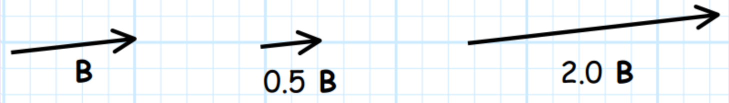


Operasi Vektor

Perkalian Vektor dengan Scalar

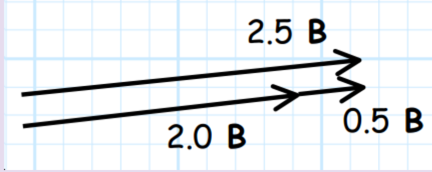
Perkalian **skalar** dan **vektor** hasilnya adalah **vektor!**

$$a\vec{B} = \vec{C} = a\left|\vec{B}\right|\hat{a}_B$$

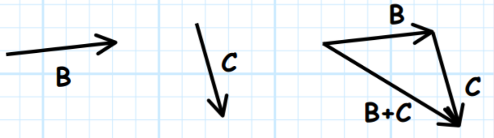


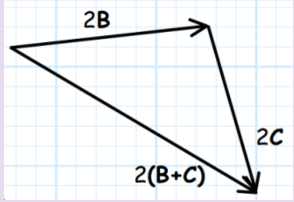
1. scalar-vector multiplication is **distributive**:

a **B**+ b **B** = (a+b) **B**



a **B**+ a **C**= a (**B**+**C**)



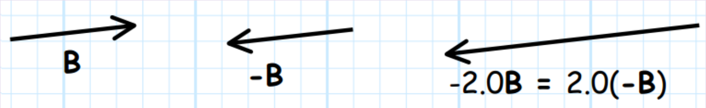


2. scalar-vector multiplication is **Commutative**:

a**B** = **B** a

3. Multiplication of a vector by a **negative scalar**:

-a **B** = a(-**B**)



4. Division of a vector by a scalar is the same as multiplying the vector by the **inverse of the scalar**

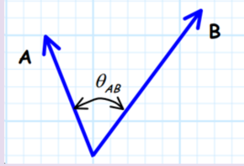
$$\frac{\vec{B}}{a} = \left(\frac{1}{a}\right)\vec{B}$$

Operasi Vektor

Dot Product / Perkalian Skalar

The dot product of two vectors is defined as:

$$\mathbf{A} \cdot \mathbf{B} = |\mathbf{A}||\mathbf{B}| \cos\theta_{AB}$$



$$0 \le \theta_{AB} \le \pi$$

IMPORTANT NOTE: The dot product is an operation involving **two vectors**, but the result is a **scalar**

1. the dot product is **commutative**

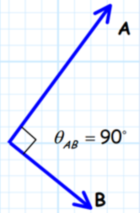
$$\vec{A} \bullet \vec{B} = \vec{B} \bullet \vec{A}$$

2. The dot product is **distributive** with addition

$$\vec{A} \bullet \left(\vec{B} + \vec{C}\right) = \vec{A} \bullet \vec{B} + \vec{A} \bullet \vec{C}$$

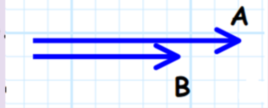
3. $\vec{A} \bullet \vec{A} = |A||A| \cos 0° = |A|^2$

4.



$$\vec{A} \bullet \vec{B} = 0$$

5.



$$\vec{A} \bullet \vec{B} = |\vec{A}||\vec{B}| \cos 0° = |\vec{A}||\vec{B}|$$

6.

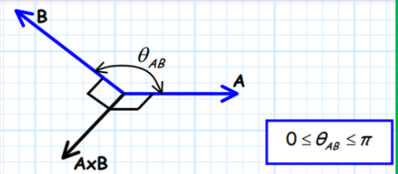
$$\vec{A} \bullet \vec{B} = |\vec{A}||\vec{B}| \cos 180° = -|\vec{A}||\vec{B}|$$

Operasi Vektor

Cross Product

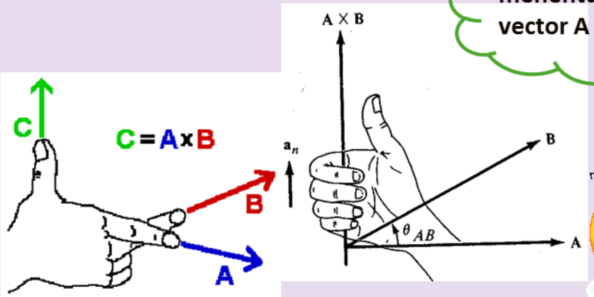
the product ofa **scalar and a vector**—is a **vector!**

$$\mathbf{A}\times\mathbf{B} = \hat{a}_n |\mathbf{A}||\mathbf{B}| \sin\theta_{AB}$$

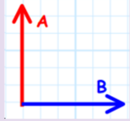


IMPORTANT NOTE: The cross product is an operation involving **two vectors**, and the result is also a **vector**

Bagaimana menentukan arah vector A x B ???

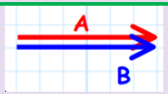


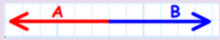
1.



$$\vec{A}\times\vec{B} = \hat{a}_n |\vec{A}||\vec{B}| \sin 90^\circ = \hat{a}_n |\vec{A}||\vec{B}|$$

2.





$$\vec{A}\times\vec{B} = \hat{a}_n |\vec{A}||\vec{B}| \sin 0^\circ = \hat{a}_n |\vec{A}||\vec{B}| \sin 180^\circ = 0$$

3. The cross product is **not commutative**

$$\vec{A}\times\vec{B} \ne \vec{B}\times\vec{A}$$

4. The cross product is also **not associative**

$$\left(\vec{A}\times\vec{B}\right)\times\vec{C} \ne \vec{A}\times\left(\vec{B}\times\vec{C}\right)$$

5. the cross product is **distributive**

$$\vec{A}\times\left(\vec{B}+\vec{C}\right) = \left(\vec{A}\times\vec{B}\right) + \left(\vec{A}\times\vec{C}\right)$$

Operasi Vektor

Triple Product

The triple product is simply a **combination** of the **dot** and **cross products.**

$$\vec{A} \bullet \vec{B} \times \vec{C} = \vec{A} \bullet \left(\vec{B} \times \vec{C} \right)$$

IMPORTANT NOTE: The triple product $\vec{A} \bullet \vec{B} \times \vec{C}$ results in a **scalar** value.

Operasi Vektor

Latihan

Let’s test your vector algebraic skills!
Can you evaluate the following expressions, and determine whether the result is a scalar (S), a vector(V), or neither (N) ??

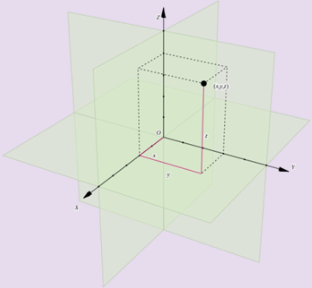
1. $(A \cdot B)C$ ________
2. $A + (B \cdot C)$ ________
3. $A \cdot (B \cdot C)$ ________
4. $A(BxC)$ ________
5. $B(A \cdot C) - C(A \cdot B)$ ________
6. $A \cdot (BxC) + C \cdot (A + B)$ ________
7. $A \cdot BxC \cdot D$ ________

Operasi Vektor

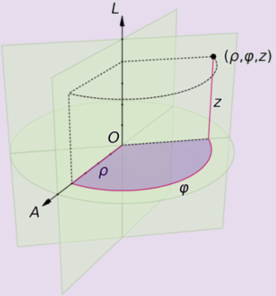
Coordinates system

- A set of 3 scalar values that define position and a set of 3 unit vectors that define direction form a **Coordinate system**
- The 3 scalar values used to define position are called **coordinates**
- All coordinates are defined with respect to an arbitrary point called the **origin**.
- The 3 unit vectors used to define direction are called **base vectors**

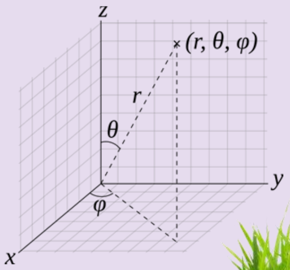
Cartesian Coordinate



Cylindrical Coordinate

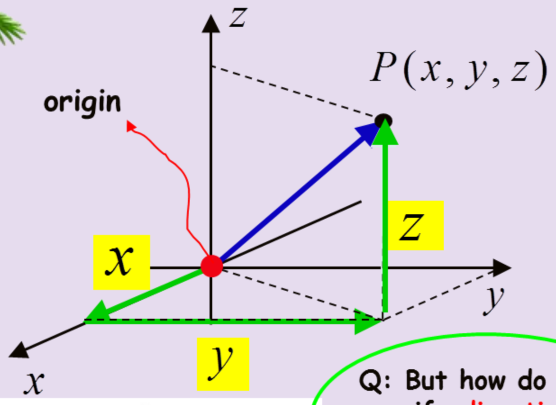


Spherical Coordinate



Operasi Vektor

Cartesian Coordinates system



- Kita dapat menentukan posisi (koordinat) **titik P** dengan 3 scalar values yaitu x, y, dan z
- coordinate values in the Cartesian system effectively represent the distance from a plane intersecting the origin
  - to specify the direction of a vector quantity is by using a well defined orthornormal set of vectors known as **base vectors**. $\hat{a}_x,\ \hat{a}_y,\ \hat{a}_z$
  - Orthonormal :

    a. Each vector is a unit vector:

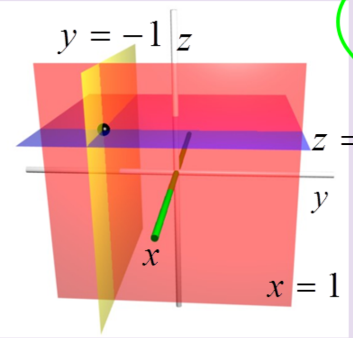
    $$\hat{a}_x \bullet \hat{a}_x = \hat{a}_y \bullet \hat{a}_y = \hat{a}_z \bullet \hat{a}_z = 1$$

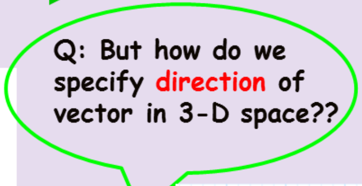
    a. Each vector is mutually orthogonal:

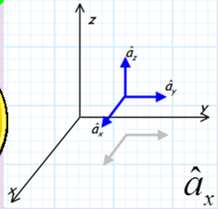
    $$\hat{a}_x \bullet \hat{a}_y = \hat{a}_y \bullet \hat{a}_z = \hat{a}_x \bullet \hat{a}_z = 0$$

  - **Additionally**,a set of base vectors must be arranged such that:

$$\hat{a}_x \times \hat{a}_y = \hat{a}_z;\ \hat{a}_y \times \hat{a}_z = \hat{a}_x;\ \hat{a}_z \times \hat{a}_x = \hat{a}_y$$



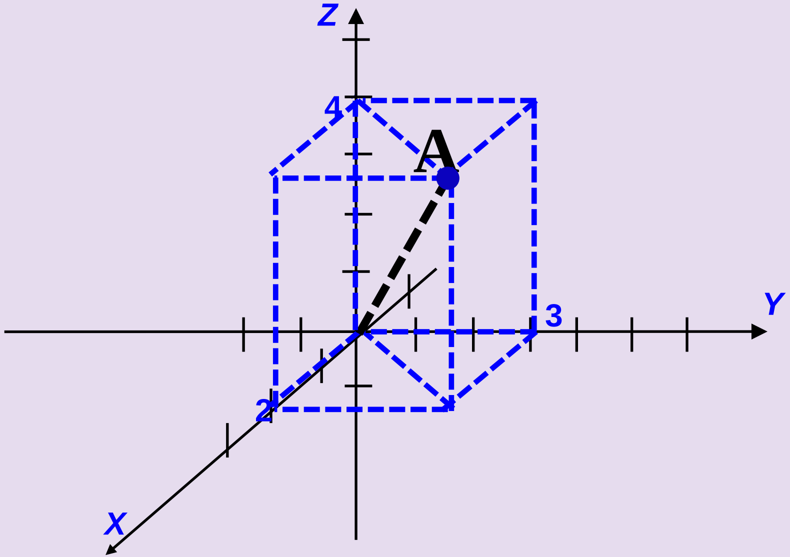




Operasi Vektor

Posisi/coordinate suatu titik pada sistem koordinat Kartesian

Titik **A** berposisi pada x=2, y=3, dan z=4 atau **A(2,3,4)** pada koordinat kertesian



Operasi Vektor

Vektor pada sistem koordinat Kartesian

$$\vec{A} = A_x \hat{a}_x + A_y \hat{a}_y + A_z \hat{a}_z$$

- Each of these three vectors point in one of the three orthogonal directions
- The magnitude of the vector is determined by the scalar values Ax, Ay, and Az that called the **scalar components** of vector A.
- The vectors $A_x \hat{a}_x, A_y \hat{a}_y, A_z \hat{a}_z$ are called the **vector components** of A.

- Menentukan Magnitude Vector:

$$\left|\vec{A}\right| = \sqrt{A_x{}^2 + A_y{}^2 + A_z{}^2}$$

- Menentukan vektor satuan searah vektor Ā

$$\hat{a}_A = \frac{\vec{A}}{\left|\vec{A}\right|} = \frac{A_x \hat{a}_x + A_y \hat{a}_y + A_z \hat{a}_z}{\sqrt{A_x{}^2 + A_y{}^2 + A_z{}^2}}$$

Operasi Vektor

Contoh

$$\vec{A} = 2\hat{a}_x + 1{,}5\hat{a}_y$$

$$\left|\vec{A}\right| = \sqrt{A_x{}^2 + A_y{}^2 + A_z{}^2}$$

$$= \sqrt{2^2 + 1{,}5^2} = \sqrt{6{,}25} = 2{,}5$$

$$\hat{a}_A = \frac{\vec{A}}{\left|\vec{A}\right|} = \frac{2\hat{a}_x + 1{,}5\hat{a}_y}{2{,}5} = 0{,}8\hat{a}_x + 0{,}6\hat{a}_y$$

Operasi Vektor

Latihan

1. Gambarkan vektor berikut dalam sistem koordinat Kartesian
   a) $\mathbf{A} = 3\mathbf{a}_x + 2\mathbf{a}_y + \mathbf{a}_z$ berpangkal di M(2,0,0)
   b) $\mathbf{B} = 2\mathbf{a}_x - \mathbf{a}_z$ berpangkal di N(0,0,2)

Operasi Vektor

Operasi Vektor pada Sistem Koordinat

Misalnya kita memiliki dua buah vektor pada koordinat kartesian :

$$\mathbf{A} = A_x\,\hat{a}_x + A_y\,\hat{a}_y + A_z\,\hat{a}_z$$
$$\mathbf{B} = B_x\,\hat{a}_x + B_y\,\hat{a}_y + B_z\,\hat{a}_z$$

Addition and Subtraction

$$\begin{aligned}\mathbf{A}+\mathbf{B} &= \left(A_x\,\hat{a}_x + A_y\,\hat{a}_y + A_z\,\hat{a}_z\right) + \left(B_x\,\hat{a}_x + B_y\,\hat{a}_y + B_z\,\hat{a}_z\right)\\ &= A_x\,\hat{a}_x + B_x\,\hat{a}_x + A_y\,\hat{a}_y + B_y\,\hat{a}_y + A_z\,\hat{a}_z + B_z\,\hat{a}_z\\ &= (A_x + B_x)\,\hat{a}_x + \left(A_y + B_y\right)\hat{a}_y + (A_z + B_z)\,\hat{a}_z\end{aligned}$$

$$\begin{aligned}\mathbf{A}-\mathbf{B} &= \left(A_x\,\hat{a}_x + A_y\,\hat{a}_y + A_z\,\hat{a}_z\right) - \left(B_x\,\hat{a}_x + B_y\,\hat{a}_y + B_z\,\hat{a}_z\right)\\ &= A_x\,\hat{a}_x - B_x\,\hat{a}_x + A_y\,\hat{a}_y - B_y\,\hat{a}_y + A_z\,\hat{a}_z - B_z\,\hat{a}_z\\ &= (A_x - B_x)\,\hat{a}_x + \left(A_y - B_y\right)\hat{a}_y + (A_z - B_z)\,\hat{a}_z\end{aligned}$$

Vektor-scalar multiplication

$$\begin{aligned}a\mathbf{B} &= a\left(B_x\,\hat{a}_x + B_y\,\hat{a}_y + B_z\,\hat{a}_z\right)\\ &= aB_x\,\hat{a}_x + aB_y\,\hat{a}_y + aB_z\,\hat{a}_z\\ &= (aB_x)\,\hat{a}_x + \left(aB_y\right)\hat{a}_y + (aB_z)\,\hat{a}_z\end{aligned}$$

Operasi Vektor

Operasi Vektor pada Sistem Koordinat

Misalnya kita memiliki dua buah vektor pada koordinat kartesian :

$$\mathbf{A} = A_x\,\hat{a}_x + A_y\,\hat{a}_y + A_z\,\hat{a}_z$$

$$\mathbf{B} = B_x\,\hat{a}_x + B_y\,\hat{a}_y + B_z\,\hat{a}_z$$

Dot Product

$$\begin{aligned}
\mathbf{A}\cdot\mathbf{B} &= \left(A_x\,\hat{a}_x + A_y\,\hat{a}_y + A_z\,\hat{a}_z\right)\cdot\left(B_x\,\hat{a}_x + B_y\,\hat{a}_y + B_z\,\hat{a}_z\right) \\
&= A_x\,\hat{a}_x\cdot\left(B_x\,\hat{a}_x + B_y\,\hat{a}_y + B_z\,\hat{a}_z\right) \\
&\quad + A_y\,\hat{a}_y\cdot\left(B_x\,\hat{a}_x + B_y\,\hat{a}_y + B_z\,\hat{a}_z\right) \\
&\quad + A_z\,\hat{a}_z\cdot\left(B_x\,\hat{a}_x + B_y\,\hat{a}_y + B_z\,\hat{a}_z\right) \\
&= A_x\,B_x\left(\hat{a}_x\cdot\hat{a}_x\right) + A_x\,B_y\left(\hat{a}_x\cdot\hat{a}_y\right) + A_x\,B_z\left(\hat{a}_x\cdot\hat{a}_z\right) \\
&\quad + A_y\,B_x\left(\hat{a}_y\cdot\hat{a}_x\right) + A_y\,B_y\left(\hat{a}_y\cdot\hat{a}_y\right) + A_y\,B_y\left(\hat{a}_y\cdot\hat{a}_z\right) \\
&\quad + A_z\,B_x\left(\hat{a}_z\cdot\hat{a}_x\right) + A_z\,B_y\left(\hat{a}_z\cdot\hat{a}_y\right) + A_z\,B_z\left(\hat{a}_z\cdot\hat{a}_z\right) = A_xB_x + A_yB_y + A_zB_z
\end{aligned}$$

Operasi Vektor

Operasi Vektor pada Sistem Koordinat

Misalnya kita memiliki dua buah vektor pada koordinat kartesian :

$$\mathbf{A} = A_x\,\hat{a}_x + A_y\,\hat{a}_y + A_z\,\hat{a}_z$$

$$\mathbf{B} = B_x\,\hat{a}_x + B_y\,\hat{a}_y + B_z\,\hat{a}_z$$

$$\vec{A}\times\vec{B} = \begin{bmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{bmatrix} \begin{matrix} \hat{a}_x & \hat{a}_y \\ A_x & A_y \\ B_x & B_y \end{matrix}$$

Cross Product

$$\begin{aligned}
\mathbf{A}\times\mathbf{B} &= \left(A_x\,\hat{a}_x + A_y\,\hat{a}_y + A_z\,\hat{a}_z\right)\times\left(B_x\,\hat{a}_x + B_y\,\hat{a}_y + B_z\,\hat{a}_z\right) \\
&= A_x\,\hat{a}_x \times\left(B_x\,\hat{a}_x + B_y\,\hat{a}_y + B_z\,\hat{a}_z\right) \\
&\quad + A_y\,\hat{a}_y \times\left(B_x\,\hat{a}_x + B_y\,\hat{a}_y + B_z\,\hat{a}_z\right) \\
&\quad + A_z\,\hat{a}_z \times\left(B_x\,\hat{a}_x + B_y\,\hat{a}_y + B_z\,\hat{a}_z\right) \\
&= A_xB_x\left(\hat{a}_x\times\hat{a}_x\right) + A_xB_y\left(\hat{a}_x\times\hat{a}_y\right) + A_xB_z\left(\hat{a}_x\times\hat{a}_z\right) \\
&\quad + A_yB_x\left(\hat{a}_y\times\hat{a}_x\right) + A_yB_y\left(\hat{a}_y\times\hat{a}_y\right) + A_yB_z\left(\hat{a}_y\times\hat{a}_z\right) \\
&\quad + A_zB_x\left(\hat{a}_z\times\hat{a}_x\right) + A_zB_y\left(\hat{a}_z\times\hat{a}_y\right) + A_zB_z\left(\hat{a}_z\times\hat{a}_z\right)
\end{aligned}$$

$$\mathbf{A}\times\mathbf{B} = \left(A_yB_z - A_zB_y\right)\hat{a}_x + \left(A_zB_x - A_xB_z\right)\hat{a}_y + \left(A_xB_y - A_yB_x\right)\hat{a}_z$$

Operasi Vektor

Latihan Soal

1. Diberikan tiga vektor pada sistem koordinat Kartesian dibawah ini :

$$\vec{A} = \hat{a}_x + \hat{a}_y$$

$$\vec{B} = \hat{a}_x + 2\hat{a}_y - 2\hat{a}_z$$

$$\vec{C} = \hat{a}_y + 2\hat{a}_z$$

tentukan hasil dari operasi-operasi vektor dibawah ini :

$a)\ \vec{A} + \vec{B}$

$b)\ \vec{B} - \vec{C}$

$c)\ 4\vec{C}$

$d)\ \vec{A} \bullet \vec{B}$

$e)\ \vec{A} \times \vec{B}$

$f)\ \vec{A} \bullet \vec{B} \times \vec{C}$

Thank you!