DTG3F3
Teknik Antena
dan propagasi

Impedansi Sendiri dan Impedansi Gandeng Antena

By : Dwi Andi Nurmantris

Contents

1 Pendahuluan

2 Impedansi Antena Linear Tipis

3 Impedansi Gandeng Antar 2 Antena

4 Pengaruh Tanah

5 Impedansi Susunan n-Elemen Identik

6

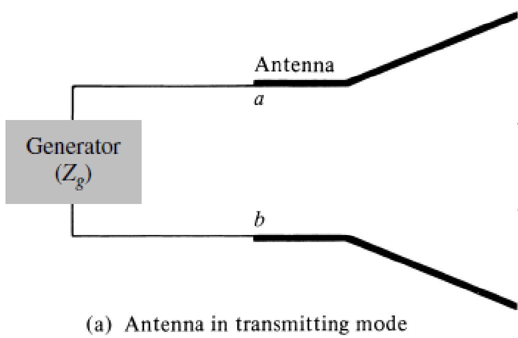
7

Where are We ?

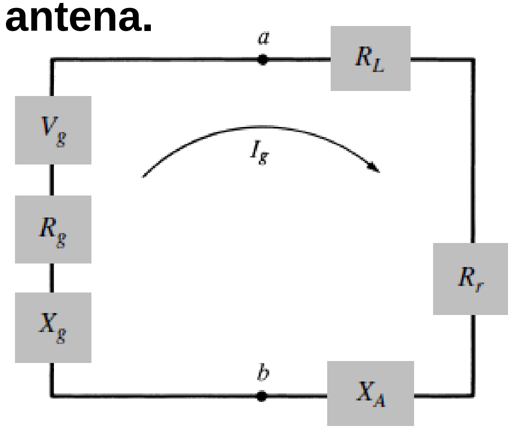
1. PENDAHULUAN
2. Jenis-jenis antenna feedline
3. Feedline pada antena tunggal dan array
4. Matching impedance
5. Balun
6.
7.

Pendahuluan

- ***Impedansi Antena/Impedansi Input Antena/Impedansi Terminal antena/Driving Point Impedance didefinisikan sebagai "Impedansi yang muncul diakibatkan oleh antena dan diukur pada terminal antena tersebut"***
- ***Impedansi Antena/Impedansi Input Antena/Impedansi Terminal antena/Driving Point Impedance*** **juga didefinisikan sebagai Rasio/perbandingan antara Voltage dengan Arus pada suatu terminal/ titik di suatu antena. Atau rasio/perbandingan antara medan listrik dan medan magnet pada suatu titik di antena.**



(a) Antenna in transmitting mode



$$Z_A = R_A + JX_A$$

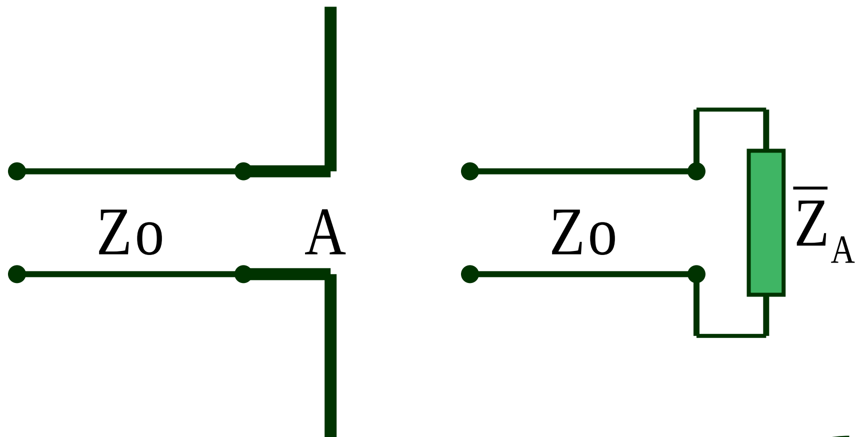
$$R_A = R_r + R_L$$

ZA = antenna impedance at terminals a –b (ohms)
RA = antenna resistance at terminals a –b (ohms)
XA = antenna reactance at terminals a –b (ohms)
Rr = radiation resistance of the antenna
RL = loss resistance (Ohmic Resistance) of the antenna

Pendahuluan

- ***Impedansi Antena/Impedansi Input Antena/Impedansi Terminal antena/Driving Point Impedance dipengaruhi oleh :***
  - ***Frekuensi Operasi***
  - ***Geometri Antena***
  - ***Metoda Pencatuan***
  - ***Kondisi Objek-objek Sekitar***
- **Metode analisis/perhitungan Impedansi Antena :**
  - ***Boundary-Value Method***
  - ***Transmission Line Method***
  - ***Poynting Vector Method***
  - ***Integral Equation Method of Moment***
  - ***Induced Emf Method (1932)***
- **Karena banyaknya variasi dan Kompleksnya geometri antena, maka hanya beberapa antena saja yang sudah dianalisa. Sedangkan untuk antena yang lain, impedansi antena hanya bisa ditentukan melalui *experimental***

Pendahuluan



- **Dari sisi saluran transmisi, antena dipandang sebagai jaringan 2 terminal yang disebut sebagai impedansi terminal / titik catu**

Impedansi antena

= Impedansi sendiri + Impedansi gandeng

- **Impedansi Sendiri**

  Jika antena terisolasi dari keadaan sekelilingnya

- **Impedansi Gandeng**

  Jika terdapat ‘benda-benda’ lain di sekitar antena dan mempengaruhi antena

Where are We ?

1. *Pendahuluan*
2. *Impedansi Antena Linear Tipis*
3. Feedline pada Antena Tunggal dan Array
4. Matching Impedance
5. Balun
6.
7.

Impedansi Sendiri Antena Linear Tipis

Metoda EMF Induksi

Kasus : Antena linear tipis dipole ½λ
Distribusi arus sinusoidal

$$
\begin{aligned}
Z_{11} &= R_{11} + j X_{11} = 30\,\mathrm{Ein}(2\pi n) \\
&= 30\left[\mathrm{Cin}(2\pi n) + j\,\mathrm{Si}(2\pi n)\right] \\
&= 30\left[0{,}577 + \ln(2\pi n) - \mathrm{Ci}(2\pi n) + j\,\mathrm{Si}(2\pi n)\right]
\end{aligned}
$$

!!

Impedansi Sendiri = (Resistansi Sendiri) + j (Reaktansi Sendiri)

dimana,

Resistansi Sendiri = R_{11} = 30 Cin (2πn)

dan,

Reaktansi Sendiri = X_{11} = 30 Si (2πn)

asumsi….

- **Arus sinusoidal**
- **L kelipatan ½λ**

Catatan :

Nilai-nilai Cin(x), Si(x) dapat dilihat pada tabel ataupun dilihat pada grafik !

Impedansi Sendiri Antena Linear Tipis

Exponential Integral

$$Ein(jy) = \int_0^{jy} \frac{1 - e^{-w}}{w} dw$$

$$Ein(jy) = Cin(y) + jSi(y)$$

$$Cin(jy) = 0{,}577 + \ln y + Ci(y)$$

Impedansi Sendiri Antena Linear Tipis

Contoh :

- **Untuk dipole ½λ ↓ n = 1**

$$\left.\begin{aligned} R_{11} &= 30\ \mathrm{Cin}\ (2\pi) = 73\ \mathrm{ohm} \\ X_{11} &= 30\ \mathrm{Si}\ (2\pi) = 45{,}5\ \mathrm{ohm} \end{aligned}\right\} \quad \underline{\underline{Z_{11} = (\ 73 + j\ 42{,}5\ )\ \mathrm{ohm}}}$$

Terlihat bahwa dipole 1/2λ memiliki sifat tidak resonan (reaktansi ≠ 0), sehingga untuk membuatnya resonan harus dipotong (1-5)%. Tindakan ini akan membuatnya resonan, tetapi resistansi sendiri dengan sendirinya juga akan berkurang dari 73 ohm

- **Untuk dipole 3/2 λ ↓ n = 3**

$$\left.\begin{aligned} R_{11} &= 30\ \mathrm{Cin}\ (6\pi) = 105{,}5\ \mathrm{ohm} \\ X_{11} &= 30\ \mathrm{Si}\ (6\pi) = 45{,}5\ \mathrm{ohm} \end{aligned}\right\} \quad \underline{\underline{Z_{11} = (\ 105{,}5 + j\ 45{,}5\ )\ \mathrm{ohm}}}$$

Impedansi Sendiri Antena Linear Tipis

Impedansi Sendiri Dipole Dengan Panjang Sembarang

(dari Proc. IRE no. 32 April 1934)

$$R_{11} = 30\left[\left(1-\cot^2\frac{\beta L}{2}\right)\mathrm{Cin}\,2\beta L + 4\cot^2\frac{\beta L}{2}\mathrm{Cin}\,\beta L + 2\cot\frac{\beta L}{2}\left(\mathrm{Si}\,2\beta L - 2\,\mathrm{Si}\,\beta L\right)\right]$$

Untuk panjang L << (kecil sekali) , dari persamaan diatas direduksi menjadi :

$$R_{11} = 5(\beta L)^2$$

Impedansi Sendiri Antena Linear Tipis

Jika antena ditempatkan di atas groundplane , dengan konduktivitas $\sigma \rightarrow \infty$, maka :

$$Z_A = \frac{1}{2} Z_{A(\text{dgn panjang} 2 \times \text{antenna tsb})}$$

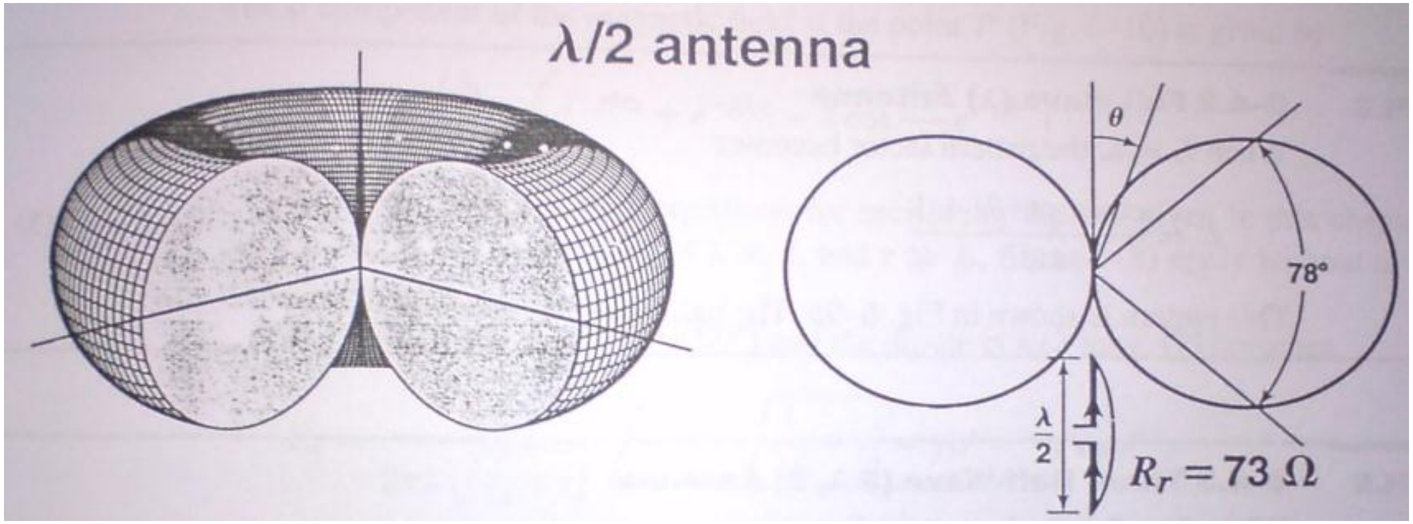
Struktur di atas disebut sebagai <u>MONOPOLE !</u>

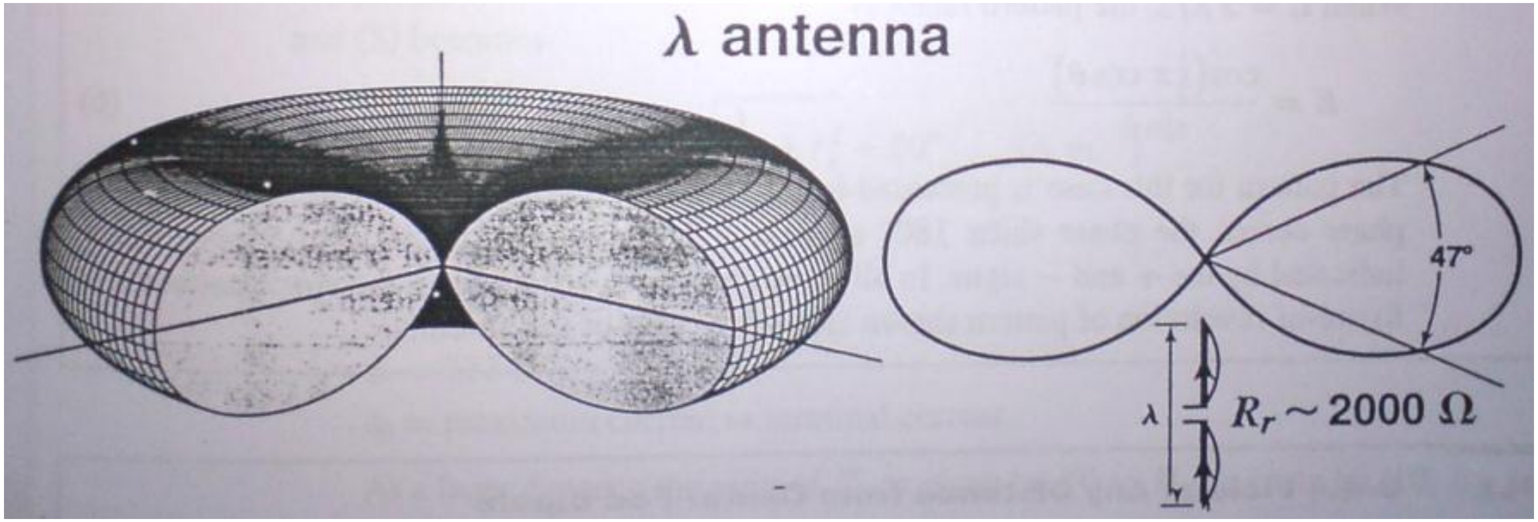
Contoh :

$$Z_{[\lambda/4]} = \frac{1}{2} \times Z_{[\lambda/2]} = (36,5 + j\,22,8) \quad \text{ohm}$$

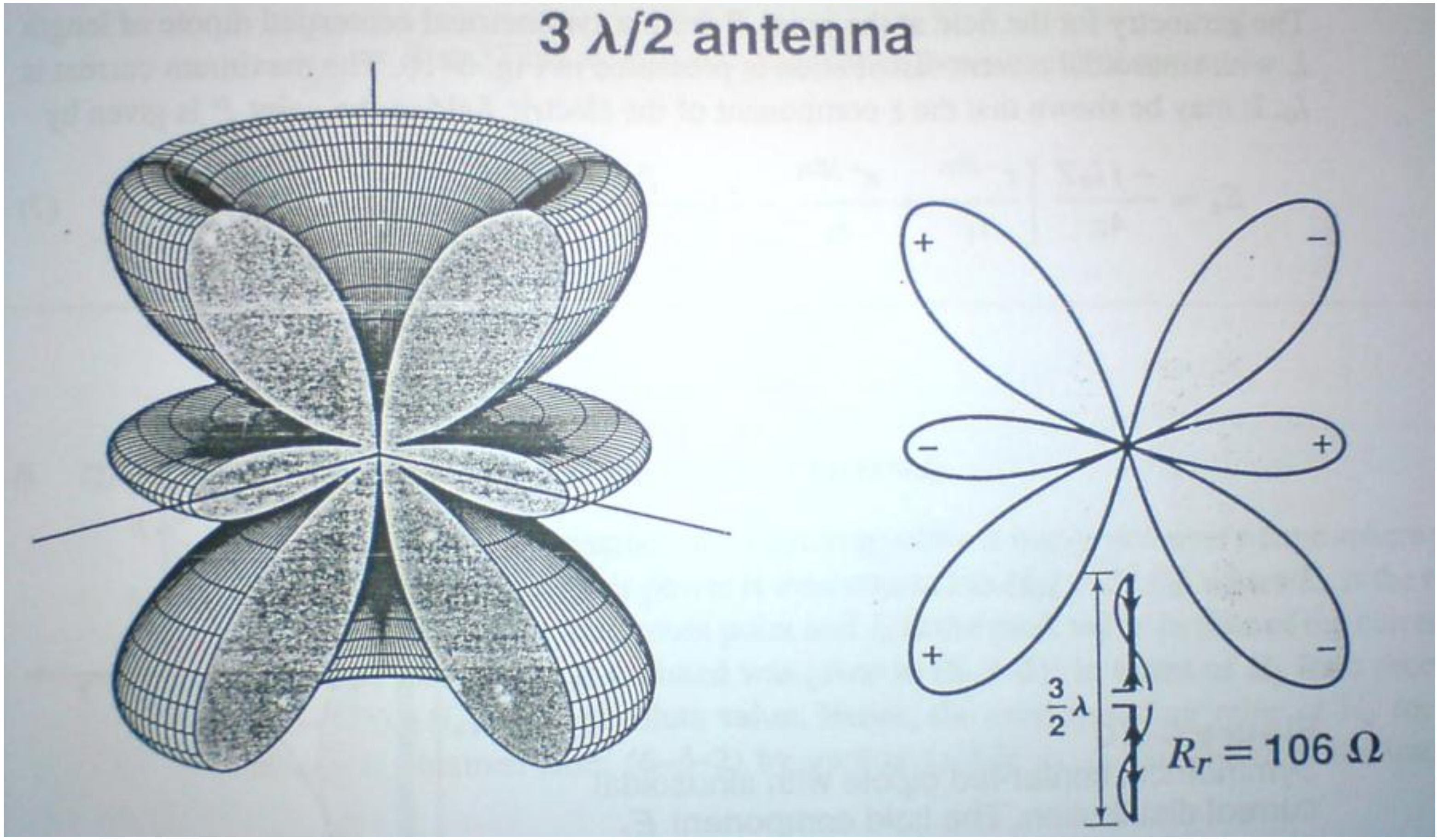
monopole $\lambda/4$ di atas groundplane

Impedansi Sendiri Antena Linear Tipis





Impedansi Sendiri Antena Linear Tipis



Where are We ?

1. Pendahuluan ✓
2. Impedansi Antena Linear Tipis ✓
3. Impedansi Gandeng Antar 2 Antena ✓
4. Matching Impedance
5. Balun
6.
7.

Impedansi Gandeng Antar dua Antena

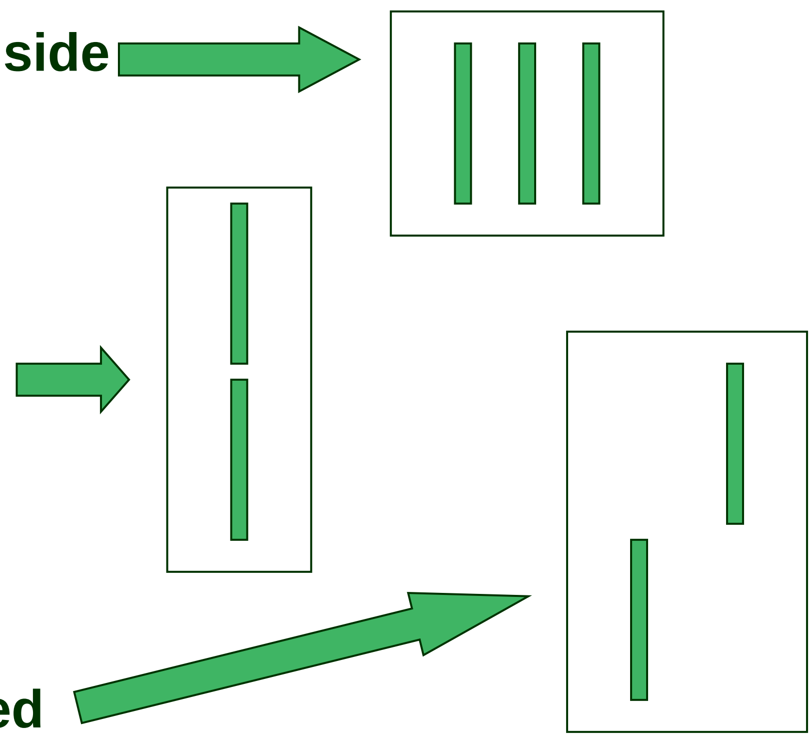
Impedansi gandeng / mutual terjadi jika terdapat 'benda-benda' (terutama konduktor) lain disekitar antena catu.

Tergantung kepada,

- **Posisi relatif antara benda tersebut dengan antena tercatu**

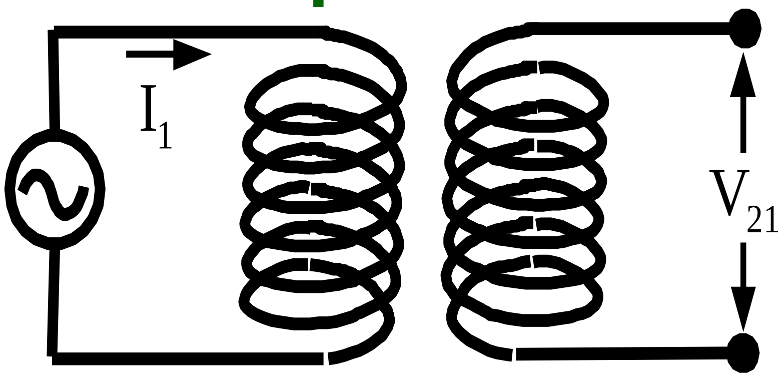
3 macam posisi relatif,

- **Side by side**
- **Kolinier**
- **Staggered**



Impedansi Gandeng Antar dua Antena

Konsep Dasar...

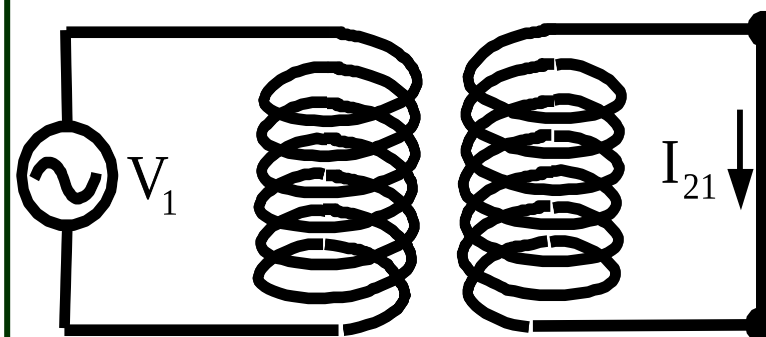


Impedansi gandeng suatu pasangan rangkaian di atas didefinisikan sebagai,

Negatif perbandingan emf induksi pada rangkaian sekunder terhadap arus primer, jika rangkaian sekunder open circuit,

$$Z_{21} = -\frac{V_{21}}{I_1}$$

Bedakan... dengan konsep impedansi transfer di bawah ini...



Pada impedansi transfer,

$$Z_{T21} = -\frac{V_1}{I_{21}}$$

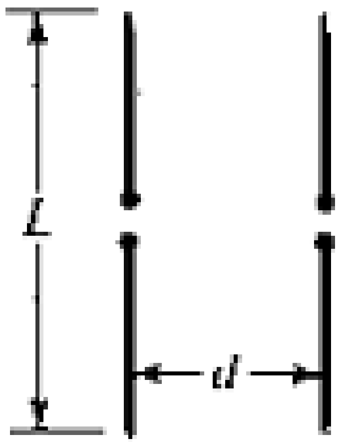
dimana,

$$Z_{T21} \neq Z_{21}$$

Side by Side...

Asumsi :

- **Panjang antena-1 sama dengan panjang antena-2 , dan merupakan kelipatan ganjil ½λ (L = n ½λ ; n ganjil)**



Side-by side

$$R_{21} = 30\left\{2Ci(\beta d) - Ci\left(\beta\left[\sqrt{d^2 + L^2} + L\right]\right) - Ci\left(\beta\left[\sqrt{d^2 + L^2} - L\right]\right)\right\}$$

$$X_{21} = -30\left\{2Si(\beta d) - Si\left(\beta\left[\sqrt{d^2 + L^2} + L\right]\right) - Si\left(\beta\left[\sqrt{d^2 + L^2} - L\right]\right)\right\}$$

Lihat di Krauss untuk penurunan lengkapnya...

Impedansi Gandeng Antar dua Antena

Impedansi Gandeng Dipole Dengan Panjang Sembarang yang disusun secara Side by Side

Oleh G.H.Brown and R.Kings "High Frequency Models in Antenna Investigation" ***Proc.IRE,22,457-480,April 1934***

$$R_{21} = 30\frac{1}{\sin^2(\beta L/2)}\Bigg\{2(2+\cos\beta L)\,\mathrm{Ci}\,\beta d$$

$$-4\cos^2\frac{\beta L}{2}\left[\mathrm{Ci}\,\frac{\beta}{2}\left(\sqrt{4d^2+L^2}-L\right)+\mathrm{Ci}\,\frac{\beta}{2}\left(\sqrt{4d^2+L^2}+L\right)\right]$$

$$+\cos\beta L\left[\mathrm{Ci}\,\beta\left(\sqrt{d^2+L^2}-L\right)+\mathrm{Ci}\,\beta\left(\sqrt{d^2+L^2}+L\right)\right]$$

$$+\sin\beta L\Bigg[\mathrm{Si}\,\beta\left(\sqrt{d^2+L^2}+L\right)-\mathrm{Si}\,\beta\left(\sqrt{d^2+L^2}-L\right)$$

$$-2\,\mathrm{Si}\,\frac{\beta}{2}\left(\sqrt{4d^2+L^2}+L\right)+2\,\mathrm{Si}\,\frac{\beta}{2}\left(\sqrt{4d^2+L^2}-L\right)\Bigg]\Bigg\}\quad(\Omega)$$

$$X_{21} = 30\frac{1}{\sin^2(\beta L/2)}\Bigg\{-2(2+\cos\beta L)\,\mathrm{Si}\,\beta d$$

$$+4\cos^2\frac{\beta L}{2}\left[\mathrm{Si}\,\frac{\beta}{2}\left(\sqrt{4d^2+L^2}-L\right)+\mathrm{Si}\,\frac{\beta}{2}\left(\sqrt{4d^2+L^2}+L\right)\right]$$

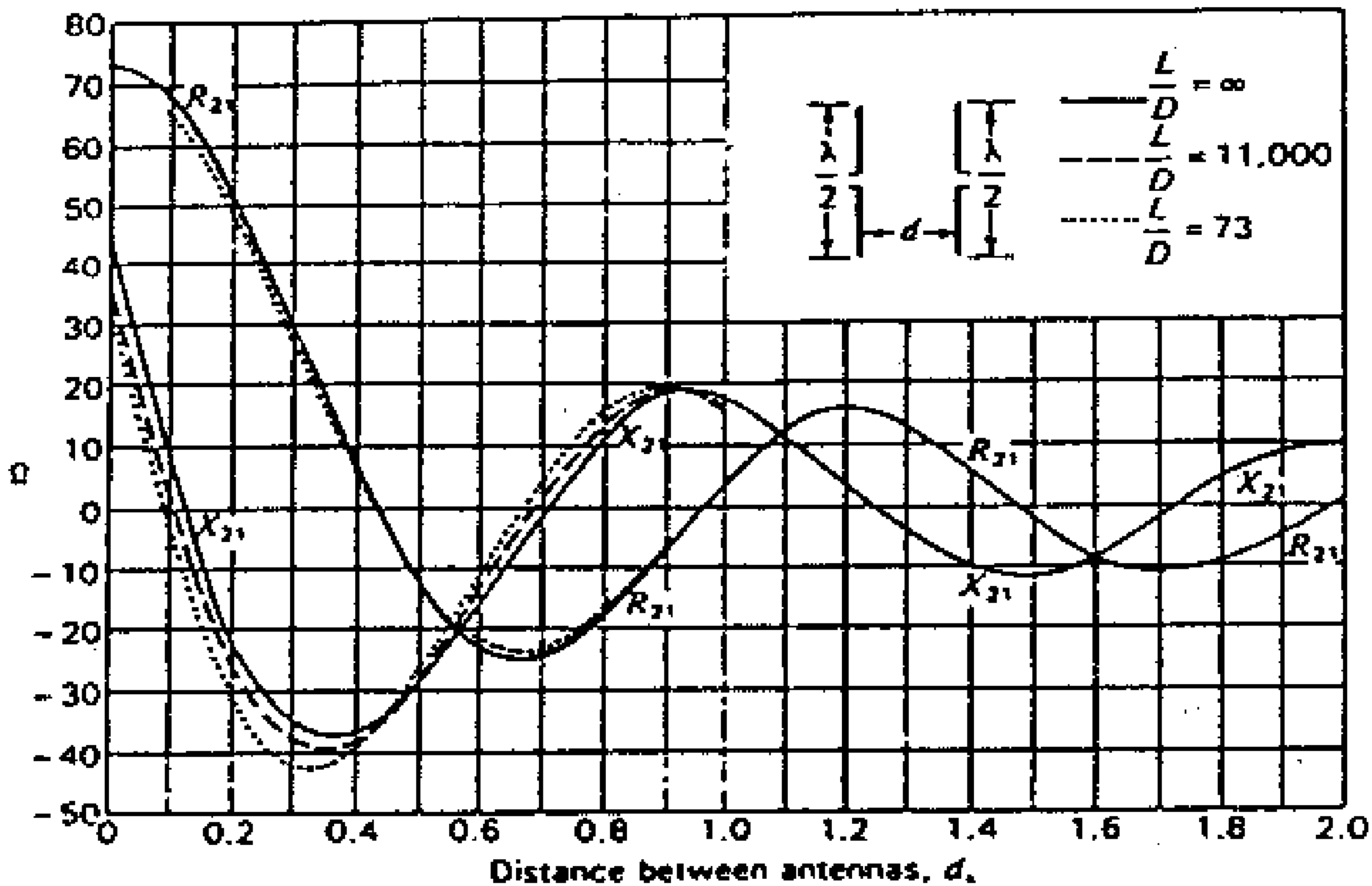
$$-\cos\beta L\left[\mathrm{Si}\,\beta\left(\sqrt{d^2+L^2}-L\right)+\mathrm{Si}\,\beta\left(\sqrt{d^2+L^2}+L\right)\right]$$

$$+\sin\beta L\Bigg[\mathrm{Ci}\,\beta\left(\sqrt{d^2+L^2}+L\right)-\mathrm{Ci}\,\beta\left(\sqrt{d^2+L^2}-L\right)$$

$$-2\,\mathrm{Ci}\,\frac{\beta}{2}\left(\sqrt{4d^2+L^2}+L\right)+2\,\mathrm{Ci}\,\frac{\beta}{2}\left(\sqrt{4d^2+L^2}-L\right)\Bigg]\Bigg\}\quad(\Omega)$$

Impedansi Gandeng Antar dua Antena

<u>Grafik resistansi dan reaktansi gandeng</u> elemen dipole λ/2 yang disusun side by side

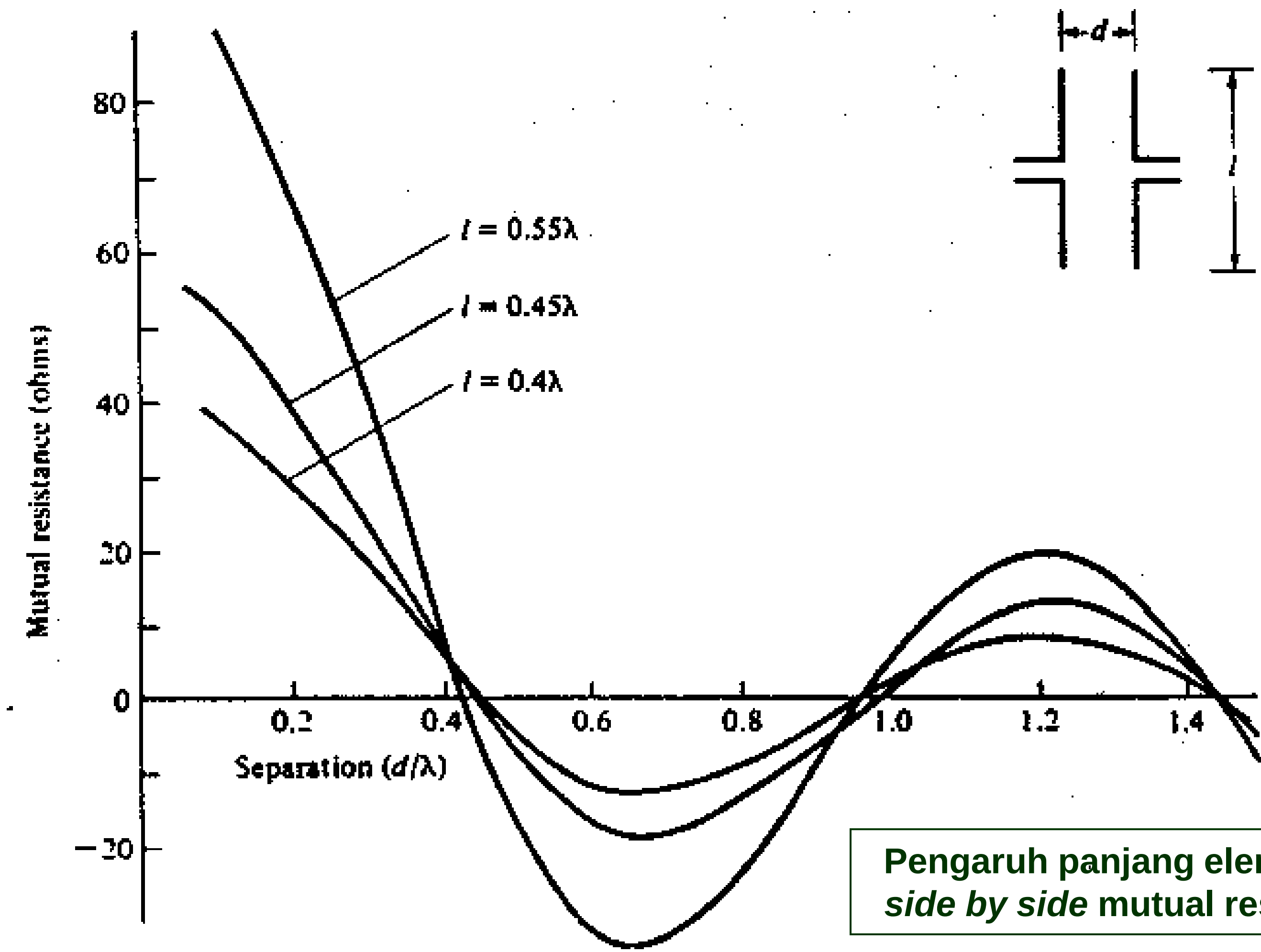


Impedansi Gandeng Antar dua Antena

Table 10-1 Mutual resistance versus spacing for thin center-fed side-by-side $\lambda/2$ antennas ($\beta L = 180°$), with sinusoidal current distribution

Spacing d	Mutual resistance R_{21}, Ω	Self minus mutual resistance $(R_{11} - R_{21})$, Ω
0.00	73.13	0.00
0.01	73.07	0.06
0.05	71.65	1.48
0.10	67.5	5.63
0.125	64.4	8.7
0.15	60.6	12.5
0.20	51.6	21.5
0.25	40.9	32.2
0.3	29.4	43.7
0.4	+6.3	66.8
0.5	−12.7	85.8
0.6	−23.4	96.5
0.7	−24.8	97.9
0.8	−18.6	91.7
0.9	−7.2	80.3
1.0	+3.8	69.3
1.1	+12.1	61.0
1.2	+15.8	57.3
1.3	+12.4	60.7
1.4	+5.8	67.3
1.5	−2.4	75.5
1.6	−8.3	81.4
1.7	−10.7	83.8
1.8	−9.4	82.5
1.9	−4.8	77.9
2.0	+1.1	72.0

Impedansi Gandeng Antar dua Antena

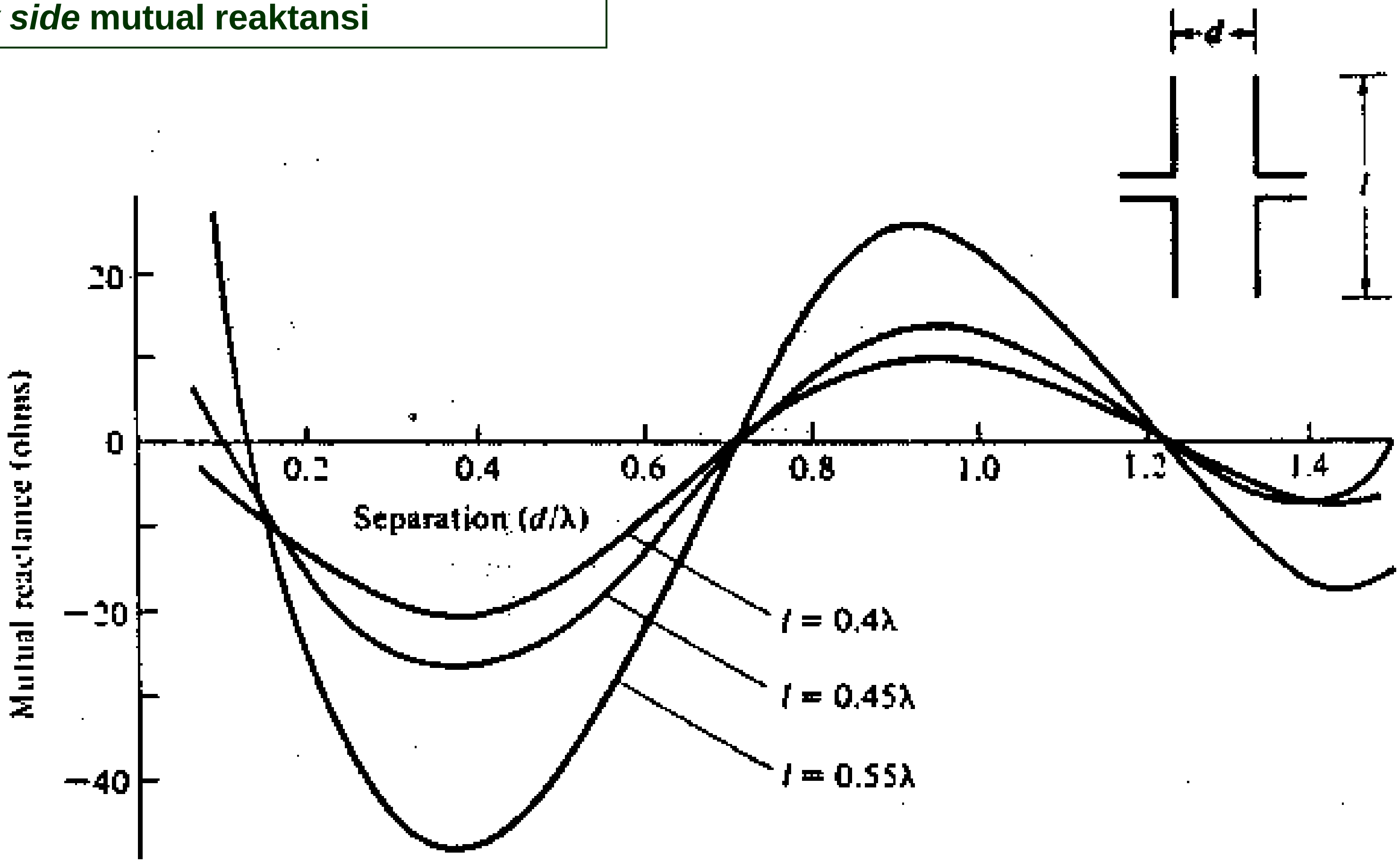


(a) Mutual resistance

Pengaruh panjang elemen thd *side by side* mutual resistansi

Impedansi Gandeng Antar dua Antena

Pengaruh panjang elemen thd *side by side* mutual reaktansi



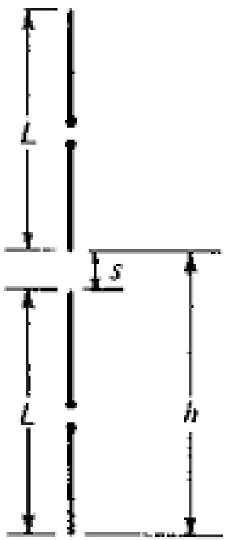
Impedansi Gandeng Antar dua Antena

Colinear...

Dengan cara yang sama, dapat diturunkan impedansi gandeng antara 2 antena yang disusun kolinier dan hasilnya adalah sbb :

$$R_{21} = -15\cos\beta h\left[2Ci\,2\beta h + Ci\,2\beta(h-L) + Ci\,2\beta(h+L) - \ln\left(\frac{h^2-L^2}{h^2}\right)\right] + 15\sin\beta h\left[2Si\,2\beta h - Si\,2\beta(h-L) - Si\,2\beta(h+L)\right]$$

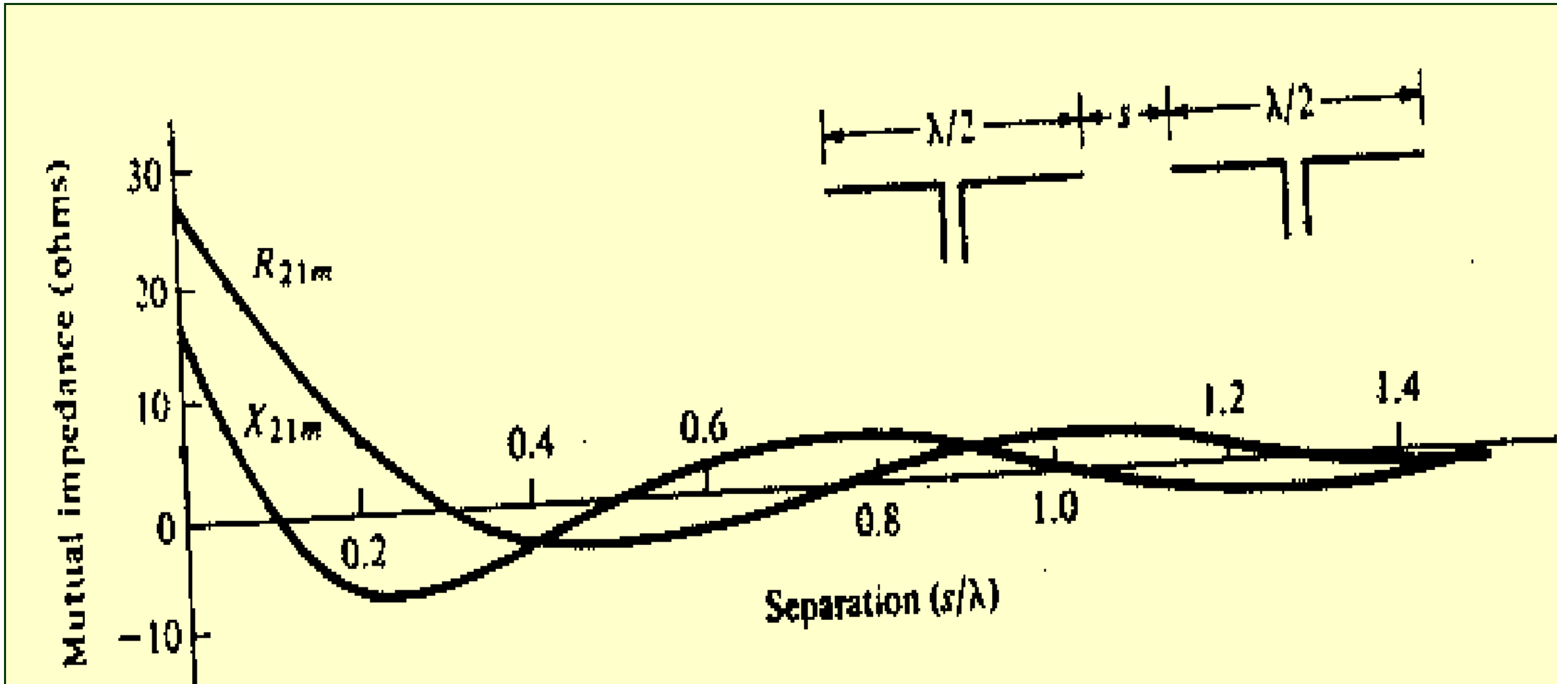
$$X_{21} = -15\cos\beta h\left[2Si\,2\beta h - Si\,2\beta(h-L) - Si\,2\beta(h+L)\right] + 15\sin\beta h\left[2Ci\,2\beta h - Ci\,2\beta(h-L) - Ci\,2\beta(h+L) - \ln\left(\frac{h^2-L^2}{h^2}\right)\right]$$



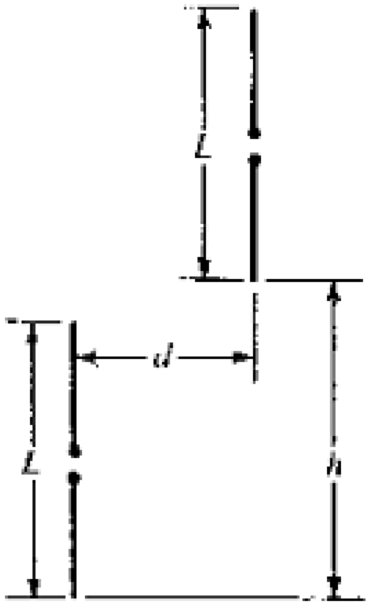
Asumsi :

- **kelipatan ganjil ½λ (L = n ½λ ; n ganjil)**
- **h>L**
- **Panjang antena-1 sama dengan panjang antena-2**

Impedansi Gandeng Antar dua Antena



Staggered / Echelon...



Staggered or in echelon

$$R_{21} = -15\cos\beta h(-2\,\text{Ci}\,A - 2\,\text{Ci}\,A' + \text{Ci}\,B + \text{Ci}\,B' + \text{Ci}\,C + \text{Ci}\,C') + 15\sin\beta h(2\,\text{Si}\,A - 2\,\text{Si}\,A' - \text{Si}\,B + \text{Si}\,B' - \text{Si}\,C + \text{Si}\,C') \quad (\Omega)$$

$$X_{21} = -15\cos\beta h(2\,\text{Si}\,A + 2\,\text{Si}\,A' - \text{Si}\,B - \text{Si}\,B' - \text{Si}\,C - \text{Si}\,C') + 15\sin\beta h(2\,\text{Ci}\,A - 2\,\text{Ci}\,A' - \text{Ci}\,B + \text{Ci}\,B' - \text{Ci}\,C + \text{Ci}\,C')$$

$$A = \beta(\sqrt{d^2 + h^2} + h)$$
$$A' = \beta(\sqrt{d^2 + h^2} - h)$$
$$B = \beta[\sqrt{d^2 + (h-L)^2} + (h-L)]$$
$$B' = \beta[\sqrt{d^2 + (h-L)^2} - (h-L)]$$
$$C = \beta[\sqrt{d^2 + (h+L)^2} + (h+L)]$$
$$C' = \beta[\sqrt{d^2 + (h+L)^2} - (h+L)]$$

Asumsi :

- **kelipatan ganjil ½λ (L = n ½λ ; n ganjil)**
- **Panjang antena-1 sama dengan panjang antena-2**

Impedansi Gandeng Antar dua Antena

Staggered / Echelon...

Table 10-2 Mutual resistance as a function of *d* and *h* (Fig. 10-15) for thin λ/2 antennas in echelon

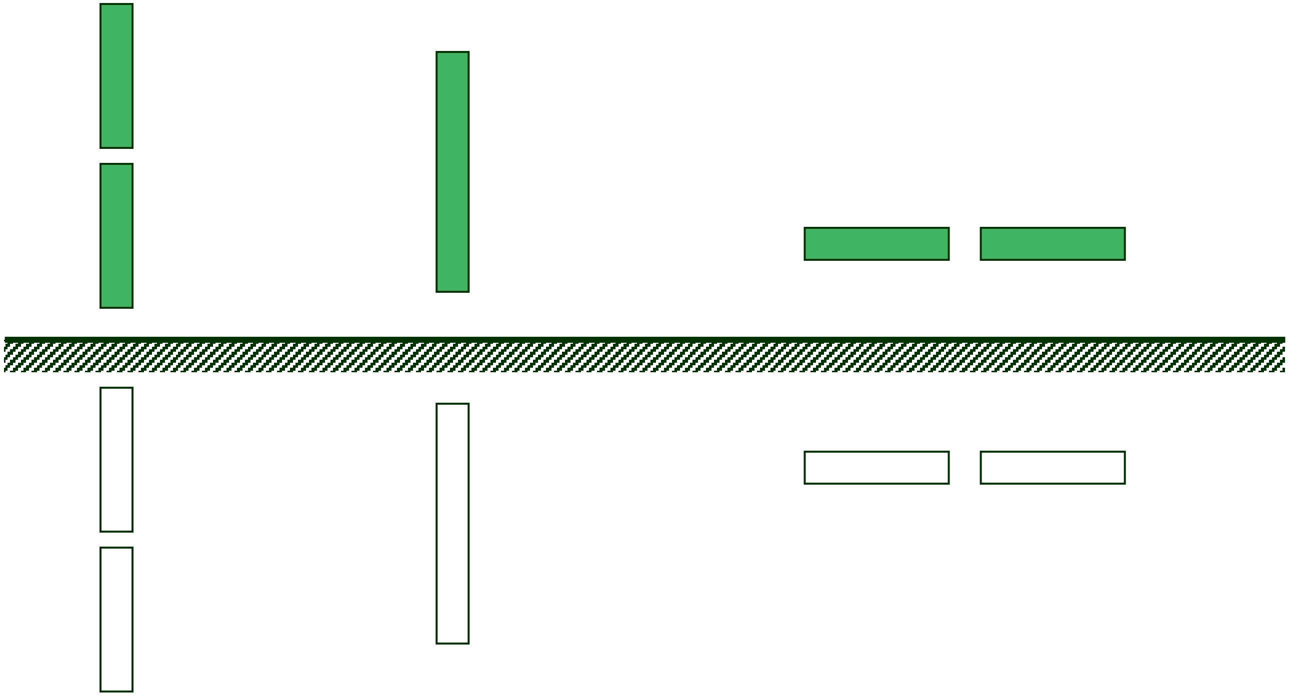
	Spacing *h*						
Spacing *d*	0.0λ	0.5λ	1.0λ	1.5λ	2.0λ	2.5λ	3.0λ
0.0λ	+73.1	+26.4	−4.1	+1.8	−1.0	+0.6	−0.4
0.5λ	−12.7	−11.8	−0.8	+0.8	−1.0	+0.5	−0.3
1.0λ	+3.8	+8.8	+3.6	−2.9	+1.1	−0.4	+0.1
1.5λ	−2.4	−5.8	−6.3	+2.0	+0.6	−1.0	+0.9
2.0λ	+1.1	+3.8	+6.1	+0.2	−2.6	+1.6	−0.5
2.5λ	−0.8	−2.8	−5.7	−2.4	+2.7	−0.3	−0.1
3.0λ	+0.4	+1.9	+4.5	+3.2	−2.1	−1.6	+1.7
3.5λ	−0.3	−1.5	−3.9	−3.8	+0.7	+2.7	−1.0
4.0λ	+0.2	+1.1	+3.1	+3.7	+0.5	−2.5	−0.1
4.5λ	−0.2	−0.9	−2.5	−3.4	−1.3	+2.0	+1.1
5.0λ	+0.2	+0.7	+2.1	+3.1	+1.8	−1.4	−1.9
5.5λ	−0.1	−0.6	−1.8	−2.9	−2.2	+0.5	+1.8
6.0λ	+0.1	+0.5	+1.6	+2.6	+2.3	−0.1	−2.0
6.5λ	−0.1	−0.5	−1.2	−2.3	−2.3	−0.5	+1.7
7.0λ	+0.1	+0.4	+1.1	+2.1	+2.3	+0.9	−1.3
7.5λ	0.0	−0.3	−1.0	−1.9	−2.1	−1.0	+0.7

Where are We ?

1. Pendahuluan
2. Impedansi Antena Linear Tipis
3. Impedansi Gandeng Antar 2 Antena
4. Pengaruh Tanah
5. Balun
6.
7.

Impedansi Gandeng Antar dua Antena

Umumnya tanah akan dianggap sebagai konduktor sempurna ($\sigma \approx \infty$) dengan luas juga ∞, sehingga antena diatas tanah dapat dianggap sebagai susunan 2 antena, yaitu yang sesungguhnya dengan bayangannya

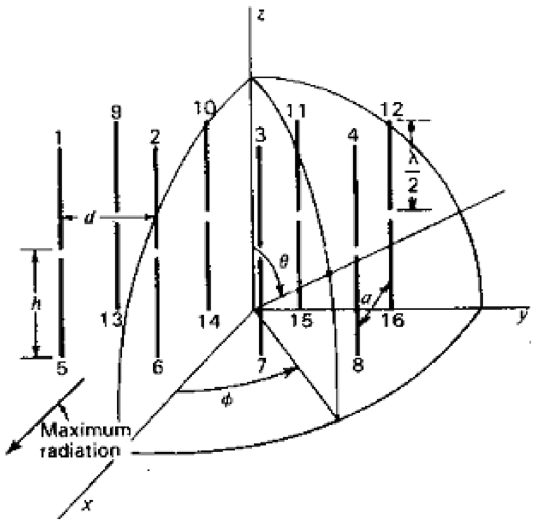


Where are We ?

1. *Pendahuluan*
2. *Impedansi Antena Linear Tipis*
3. *Impedansi Gandeng Antar 2 Antena*
4. *Pengaruh Tanah*
5. Impedansi Susunan n-Elemen Identik
6.
7.

Impedansi Gandeng Antar dua Antena

- **Hubungan-hubungan yang mendasari :**



$$V_1 = I_1 Z_{11} + I_2 Z_{12} + I_3 Z_{13} + \ldots\ldots + I_n Z_{1n}$$

$$V_2 = I_1 Z_{21} + I_2 Z_{22} + I_3 Z_{23} + \ldots\ldots + I_n Z_{2n}$$

$$V_3 = I_1 Z_{31} + I_2 Z_{32} + I_3 Z_{33} + \ldots\ldots + I_n Z_{3n}$$

$$\vdots$$

$$\boxed{V_n = I_1 Z_{n1} + I_2 Z_{n2} + I_3 Z_{n3} + \ldots\ldots + I_n Z_{nn}}$$

dengan :

V_n = tegangan terminasi elemen ke-n
I_n = arus terminasi elemen ke-n
Z_{nn} = self-impedance elemen ke-n
Z_{ij} = impedansi gandeng antara elemen ke-i dan ke-j

- **Dapat dinyatakan dalam bentuk matriks :**

$$\boxed{[V_n] = [Z_{nn}][I_n]}$$

Impedansi Gandeng Antar dua Antena

- **Impedansi terminasi/titik catu/driving point masing-masing elemen :**

$$Z_1 = \frac{V_1}{I_1} = Z_{11} + \frac{I_2}{I_1} Z_{12} + \frac{I_3}{I_1} Z_{13} + \ldots\ldots + \frac{I_n}{I_1} Z_{1n}$$

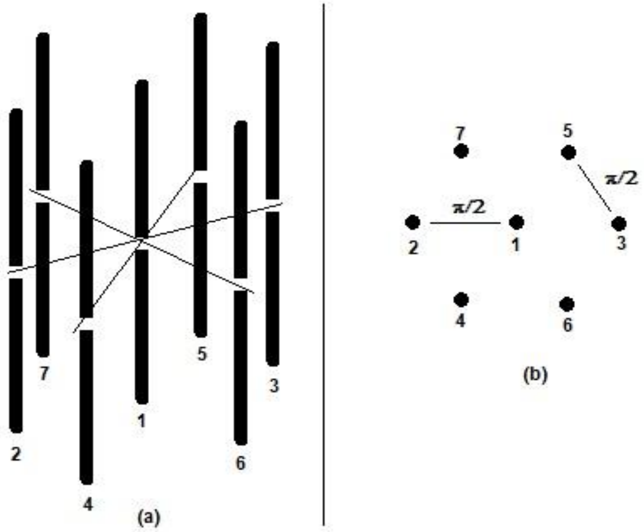
$$Z_2 = \frac{V_2}{I_2} = Z_{22} + \frac{I_1}{I_2} Z_{21} + \frac{I_3}{I_2} Z_{23} + \ldots\ldots + \frac{I_n}{I_2} Z_{2n}$$

dst

Jika arus-arus pada semua elemen, self impedances, dan mutual impedances diketahui, maka impedansi pada terminasi akan dapat dihitung !

Impedansi Gandeng Antar dua Antena

Contoh Soal



Jika diketahui jarak antena 1 ke antena 2, 3, 4, 5, 6, dan 7 adalah setengah panjang gelombang. Jarak antena 2 ke 4 yaitu setengah panjang gelombang, demikian halnya jarak antena 4 ke 6, 6 ke 3, 3 ke 5, 5 ke 7, serta 7 ke 2. Seluruh antena dicatu dengan arus yang sama, tetapi dengan beda fasa:

a. Antena 2 dan 3 terhadap antena 1 → leading 60°
b. Antena 4 dan 5 terhadap antena 1 → leading 120°
c. Antena 6 dan 7 terhadap antena 1 → leading 180°
d. Seumpama di antena 1 berfasa 0°, maka: antena 2 dan 3 berfasa 60°, antena 4 dan 5 berfasa 120°, serta antena 6 dan 7 berfasa 180°.

Tentukan Impedansi yang terukur pada masing-masing antena 1, 2, dan 5 berdasarkan formasi tersebut!

Catatan : gunakan bantuan grafik mutual impedance yang disediakan di bagian belakang soal.

x	$S_i(x)$	$C_i(x)$	$C_{in}(x)$	x	$S_i(x)$	$C_i(x)$	$C_{in}(x)$
0.0	0.0	-∞	0.0	5.0	1.54993	-0.19003	2.37647
0.1	0.09994	-1.72787	0.00228	5.1	1.53125	-0.18348	2.38972
0.2	0.19956	-1.04220	0.00977	5.2	1.51367	-0.17525	2.40091
0.3	0.29850	-0.64917	0.02220	5.3	1.49731	-0.16551	2.41021
0.4	0.39646	-0.37881	0.03952	5.4	1.48230	-0.15439	2.41778
0.5	0.49311	-0.17778	0.06164	5.5	1.46872	-0.14205	2.42380
0.6	0.58813	-0.02227	0.08845	5.6	1.45667	-0.12867	2.42844
0.7	0.68122	0.10051	0.11981	5.7	1.44620	-0.11441	2.43188
0.8	0.77209	0.19828	0.15558	5.8	1.43736	-0.09944	2.43430
0.9	0.86047	0.27607	0.19557	5.9	1.43018	-0.08393	2.43588
1.0	0.94608	0.33740	0.23960	6.0	1.42469	-0.06806	2.43682
1.1	1.02868	0.38487	0.28744	6.1	1.42087	-0.05198	2.43727
1.2	1.10805	0.42046	0.33886	6.2	1.41871	-0.03587	2.43742
1.3	1.18396	0.44574	0.39363	6.3	1.41817	-0.01989	2.43744
1.4	1.25622	0.46201	0.45146	6.4	1.41922	-0.00418	2.43748
1.5	1.32468	0.47036	0.51211	6.5	1.42179	0.01110	2.43770
1.6	1.38918	0.47173	0.57527	6.6	1.42582	0.02582	2.43824
1.7	1.44959	0.46697	0.64066	6.7	1.43120	0.03985	2.43925
1.8	1.50581	0.45681	0.70797	6.8	1.43787	0.05308	2.44084
1.9	1.55777	0.44194	0.77691	6.9	1.44570	0.06539	2.44313
2.0	1.60541	0.42298	0.84717	7.0	1.45460	0.07669	2.44621
2.1	1.64870	0.40051	0.91842	7.1	1.46443	0.08691	2.45019
2.2	1.68762	0.37508	0.99038	7.2	1.47509	0.09596	2.45512
2.3	1.72221	0.34718	1.06273	7.3	1.48644	0.10379	2.46108
2.4	1.75248	0.31729	1.13517	7.4	1.49834	0.11036	2.46812
2.5	1.77852	0.28587	1.20742	7.5	1.51068	0.11563	2.47627
2.6	1.80039	0.25334	1.27917	7.6	1.52331	0.11960	2.48555
2.7	1.81821	0.22008	1.35017	7.7	1.53611	0.12225	2.49597
2.8	1.83210	0.18649	1.42013	7.8	1.54894	0.12359	2.50754
2.9	1.84219	0.15290	1.48881	7.9	1.56167	0.12364	2.52022
3.0	1.84865	0.11963	1.55598	8.0	1.57419	0.12243	2.53401
3.1	1.85166	0.08699	1.62141	8.1	1.58637	0.12002	2.54885
3.2	1.85140	0.05526	1.68489	8.2	1.59810	0.11644	2.56469
3.3	1.84808	0.02468	1.74624	8.3	1.60928	0.11177	2.58149
3.4	1.84191	-0.00452	1.80529	8.4	1.61981	0.10607	2.59916
3.5	1.83312	-0.03213	1.86189	8.5	1.62960	0.09943	2.61763
3.6	1.82195	-0.05797	1.91591	8.6	1.63857	0.09194	2.63682
3.7	1.80862	-0.08190	1.96723	8.7	1.64665	0.08368	2.65664
3.8	1.79339	-0.10378	2.01578	8.8	1.65379	0.07476	2.67699
3.9	1.77650	-0.12350	2.06147	8.9	1.65993	0.06528	2.69777
4.0	1.75820	-0.14098	2.10427	9.0	1.66504	0.05535	2.71887
4.1	1.73874	-0.15617	2.14415	9.1	1.66908	0.04507	2.74020
4.2	1.71837	-0.16901	2.18110	9.2	1.67205	0.03456	2.76165
4.3	1.69732	-0.17951	2.21512	9.3	1.67393	0.02391	2.78310
4.4	1.67583	-0.18766	2.24626	9.4	1.67473	0.01325	2.80446
4.5	1.65414	-0.19349	2.27457	9.5	1.67446	0.00268	2.82561
4.6	1.63246	-0.19705	2.30010	9.6	1.67316	-0.00771	2.84647
4.7	1.61100	-0.19839	2.32295	9.7	1.67084	-0.01780	2.86693
4.8	1.58997	-0.19760	2.34322	9.8	1.66757	-0.02752	2.88690
4.9	1.56956	-0.19478	2.36101	9.9	1.66338	-0.03676	2.90630

x	$S_i(x)$	$C_i(x)$	$C_{in}(x)$	x	$S_i(x)$	$C_i(x)$
10.0	1.65835	-0.04546	2.92504	15.0	1.61819	0.04628
10.1	1.65253	-0.05352	2.94306	15.1	1.62226	0.04102
10.2	1.64600	-0.06089	2.96028	15.2	1.62575	0.03543
10.3	1.63883	-0.06751	2.97665	15.3	1.62865	0.02955
10.4	1.63112	-0.07332	2.99212	15.4	1.63093	0.02345
10.5	1.62294	-0.07828	3.00666	15.5	1.63258	0.01719
10.6	1.61439	-0.08237	3.02022	15.6	1.63359	0.01085
10.7	1.60556	-0.08555	3.03279	15.7	1.63396	0.00447
10.8	1.59654	-0.08781	3.04435	15.8	1.63370	-0.00187
10.9	1.58743	-0.08915	3.05491	15.9	1.63280	-0.00812
11.0	1.57831	-0.08956	3.06446	16.0	1.63130	-0.01420
11.1	1.56927	-0.08907	3.07302	16.1	1.62921	-0.02007
11.2	1.56042	-0.08769	3.08061	16.2	1.62657	-0.02566
11.3	1.55182	-0.08546	3.08726	16.3	1.62339	-0.03093
11.4	1.54356	-0.08240	3.09301	16.4	1.61973	-0.03583
11.5	1.53571	-0.07857	3.09792	16.5	1.61563	-0.04031
11.6	1.52835	-0.07401	3.10202	16.6	1.61112	-0.04433
11.7	1.52155	-0.06879	3.10538	16.7	1.60627	-0.04786
11.8	1.51535	-0.06297	3.10806	16.8	1.60111	-0.05087
11.9	1.50981	-0.05661	3.11014	16.9	1.59572	-0.05334
12.0	1.50497	-0.04978	3.11169	17.0	1.59014	-0.05524
12.1	1.50087	-0.04257	3.11277	17.1	1.58443	-0.05657
12.2	1.49755	-0.03504	3.11348	17.2	1.57865	-0.05732
12.3	1.49501	-0.02729	3.11389	17.3	1.57285	-0.05749
12.4	1.49327	-0.01938	3.11408	17.4	1.56711	-0.05708
12.5	1.49234	-0.01141	3.11414	17.5	1.56146	-0.05610
12.6	1.49221	-0.00344	3.11414	17.6	1.55597	-0.05458
12.7	1.49286	0.00443	3.11417	17.7	1.55070	-0.05252
12.8	1.49430	0.01214	3.11431	17.8	1.54568	-0.04997
12.9	1.49647	0.01961	3.11462	17.9	1.54097	-0.04694
13.0	1.49936	0.02676	3.11518	18.0	1.53661	-0.04348
13.1	1.50292	0.03355	3.11607	18.1	1.53264	-0.03962
13.2	1.50711	0.03989	3.11733	18.2	1.52909	-0.03540
13.3	1.51188	0.04574	3.11903	18.3	1.52600	-0.03088
13.4	1.51716	0.05104	3.12121	18.4	1.52339	-0.02610
13.5	1.52290	0.05576	3.12393	18.5	1.52128	-0.02111
13.6	1.52905	0.05984	3.12722	18.6	1.51969	-0.01596
13.7	1.53552	0.06327	3.13112	18.7	1.51863	-0.01071
13.8	1.54225	0.06602	3.13565	18.8	1.51810	-0.00540
13.9	1.54917	0.06806	3.14083	18.9	1.51810	-0.00010
14.0	1.55621	0.06940	3.14666	19.0	1.51863	0.00515
14.1	1.56330	0.07002	3.15316	19.1	1.51967	0.01029
14.2	1.57036	0.06993	3.16031	19.2	1.52122	0.01528
14.3	1.57733	0.06914	3.16812	19.3	1.52324	0.02006
14.4	1.58414	0.06767	3.17656	19.4	1.52572	0.02459
14.5	1.59072	0.06554	3.18561	19.5	1.52862	0.02883
14.6	1.59701	0.06278	3.19524	19.6	1.53192	0.03274
14.7	1.60296	0.05943	3.20541	19.7	1.53557	0.03628
14.8	1.60850	0.05554	3.21609	19.8	1.53954	0.03943
14.9	1.61360	0.05113	3.22723	19.9	1.54377	0.04215
				20.0	1.54824	0.04442

Questions???

Thank You !